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# BRIEF COMMUNICATION

## HOLDUP OF LIQUID DRAINING FROM INCLINED TUBES

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Spedding & Nguyen (1978) studied the flow of water down an inclined tube by measuring the liquid holdup $\bar{R}_L$ using quick-shutting valves in a 4.55 cm diameter perspex tube inclined at various angles $\alpha$ below the horizontal. Although data for $\alpha = 0°$ may be predicted using the theoretical predictions of Henderson (1966) for open channel flow, the case of $0° < \alpha \leq 90$ will be examined analytically in this work.

### ANALYSIS

The idealised situation of "free-surface channel flow" is depicted schematically in figure 1 for $0° < \alpha < 90°$, and in figure 2 for $\alpha = 90°$. The air is stationary and the liquid surface is assumed smooth. Neglecting the density of the air and the shear stress at the gas–liquid interface, a simple force balance in the liquid phase gives

$$\tau_{WL} S_L = \rho_L A_L g \sin\alpha \qquad [1]$$

where the drag due to the solid surface balances the gravitational forces (Davies 1972), $\tau_{WL}$ is the shear force between the liquid and the tube wall, $S_L$ the wetted perimeter, $\rho_L$ the liquid density, $A_L$ the area of flow and $g$ the acceleration due to gravity.

The shear force expressed in terms of the friction factor $f_L$ is

$$\tau_{WL} = \frac{1}{2} f_L \rho_L \bar{V}_L^{\,2} \qquad [2]$$

where $\bar{V}_L = Q_L/A_L = 4Q_L/\bar{R}_L\ \pi D^2$ the true flow velocity of the liquid, $Q_L$ = liquid flow rate, $\bar{R}_L$ = the liquid holdup, and $D$ = pipe diameter.

For the case of a smooth pipe surface, the friction factor may be written in the Blasius form (Chen & Spedding 1983)

$$f_L = C\,\mathrm{Re}^{-m} = C\left[\frac{\rho_L \bar{V}_L \bar{D}_L}{\mu_L}\right]^{-m} \qquad [3]$$

where Re is the Reynolds number, $\mu_L$ the viscosity, $\bar{D}_L$ the hydraulic diameter while $C$ and $m$ are constants which assume different values depending on the type of flow. For laminar flow, $\mathrm{Re} \leq 2000$, $C = 16.0$ and $m = 1.0$; for turbulent flow, $\mathrm{Re} > 2000$, $C = 0.046$ and $m = 0.2$.

For the case of $0° < \alpha < 90°$, since the hydraulic diameter is given as $\bar{D}_L = 4A_L/S_L$, in the same way as used by Chen & Spedding (1981) in analysing two-phase gas–liquid stratified flow,

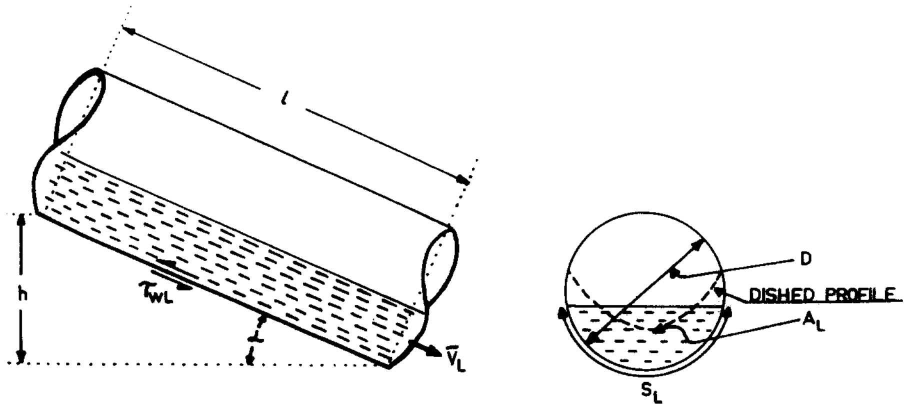


Figure 1. Ideal situation representing free surface channel flow in an inclined tube when $0° < \alpha < 90°$.

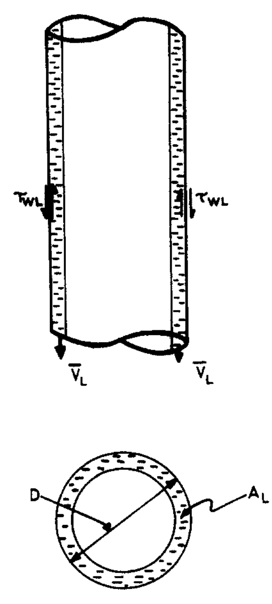


Figure 2. Ideal situation representing free surface channel flow in a vertical tube, $\alpha = 90°$.

[1], after combining with [2] and [3], gives

$$\frac{\bar{R}_L^3}{S_L^{(1+m)}} = \frac{C(\mu_L/4\rho_L)^m Q_L^{(2-m)}}{2g(\pi D^2/4)^3 \sin \alpha}. \qquad [4]$$

In the case of $\alpha = 90°$, the sin $\alpha$ term in [1] becomes unity and the hydraulic diameter becomes $\bar{D}_L = 4A_L/\pi D$ since the wetted perimeter is now the entire pipe inside circumference. As a result, [1] reduces to

$$\bar{R}_L^3 = \frac{C(\mu_L/4\rho_L)^m (\pi D)^{(1+m)} Q L^{(2-m)}}{2g(\pi D^2/4)^3}. \qquad [5]$$

Equations [4] and [5] are general. For any particular system, $\mu_L$, $\rho_L$ and $D$ are fixed, while $C$ and $m$ also are fixed for the particular type of flow. The term $S_L$ in [4] may be solved in terms of $\bar{R}_L$ as is obvious from an inspection of figure 1. Thus, [4] provides a relationship for $\bar{R}_L$ and $Q_L$ for any value of $\alpha$ in the range $0° < \alpha < 90°$, while [5] serves for the case of $\alpha = 90°$.

Kubie & Gardner (1978) obtained an equation for the steady state velocity of liquid draining in inclined tubes in relation to two-phase gas–liquid flow in $Y$-junctions. Their equation may be rearranged to give the holdup as

$$\bar{R}_L = \frac{f_L^{0.39}}{0.89\pi}\left[\frac{Q_L}{(g(D/2)^5 \sin\alpha)^{1/2}}\right]^{0.78} \qquad [6]$$

Kubie & Gardner (1978) besides using an empirical relation linking wetted perimeter to liquid flow area also used a constant value of $f_L = 0.008$ for the friction factor. In this work, the variation of friction factor with the flow Reynolds number has been allowed for by using the Blasius form of equation as shown in [3].

## COMPARISON WITH EXPERIMENTAL RESULTS

Equations [4] is solved for the conditions of the system used by Spedding & Nguyen (1978) where almost all the data had Reynolds numbers in excess of 4000 and are therefore outside of the laminar and transitional regimes. The following characteristics apply: $C = 0.046$, $m = 0.2$, $D = 4.55$ cm, $\rho_L = 1.0$ gm cm$^{-3}$ and $\mu_L = 0.01$ gm (cm s)$^{-1}$. The results are plotted in figures 3 as $\bar{R}_L$ against $Q_L$.

The experimental data show general agreement with the predictions which are obtained using the present model. However, at low $Q_L$ values, positive deviations are in evidence between the experimental data and those which are predicted for the case of $0° < \alpha < 90°$. This may be attributable partly to the difficulties which are associated with measuring low values of $\bar{R}_L$ at low liquid flow rates. Moreover, when $Q_L$ is small, liquid is flowing in a small cross-sectional area of the pipe in the lower portion of the pipe cross section. Under these circumstances the liquid path is observed to snake rather than flow in a straight path down the tube. Furthermore, the flow path is not constant but is observed to oscillate somewhat about an equilibrium position in a similar manner to the meandering of a river.

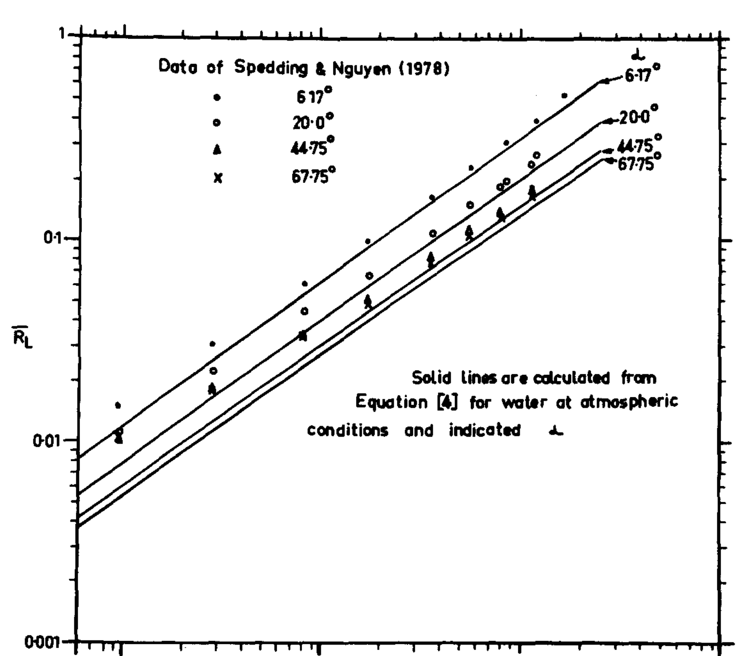


Figure 3. Comparison of predicted $\bar{R}_L$ using [4] against measured $\bar{R}_L$ at various angles of inclination and liquid rates.

The meandering of the flow increases the length of the flow path within the pipe and in so doing the liquid is swept up the curved inside of the pipe and requires a definite time to drain away when the liquid flow path changes due to the onset of oscillations. Such movement would be expected to lead to an increase in the observed holdup and thus give the positive deviation from theory which is observed at low values of $Q_L$ in figure 3.

When the tube inclination is small, at $\alpha = 6.17$, the experimental values for $\bar{R}_L$ approach the theoretical prediction above a flow rate of $Q_L = 150\ \text{cm}^3\ \text{s}^{-1}$. Coincidence is not achieved above this flow rate, however, as the experimental values are about +5% above the prediction. The basic reason why the difference between the measured and predicted holdup increases with angle of inclination for the case $0° < \alpha < 90°$, is that the liquid surface in the tube is not flat in the radial direction but is dished in the manner which is indicated by the dotted surface in figure 1. The velocity profile through the flowing liquid is such as to generate the dished surface thus causing a minor increase in the measured holdup over that predicted. Of course for the case where $\alpha = 90°$ the liquid surface in general will coincide with that of the internals of the pipe, as anticipated in the theoretical development of [5].

The predictions using [6] are plotted in figure 4 against the measured values and for ease of comparison, similar plots for [4] are given in figure 5. The predictions of [6] are lower than the actual at $\bar{R}_L$ less than 0.1 but are higher at $\bar{R}_L$ greater than about 0.1. The predictions of [4] which allow for the Reynolds number dependence of the friction factor by the use of the Blasius form of [3] consistently under predicts by about 10% over the entire range of $\bar{R}_L$ values available for $\bar{R}_L > 0.05$. If instead of using $f_L = 0.008$, the $f_L$ as given in [3] was used in [6], the resultant predictions become almost identical with those shown in figure 5. A further comparison of the experimental results of Spedding & Nguyen (1978) with [6] is shown in figure 6.

With reference to figures 3 and 6, the positive deviation between $\bar{R}_L$ data and theoretical prediction at low liquid flow rates is observed to increase with increasing pipe inclination for

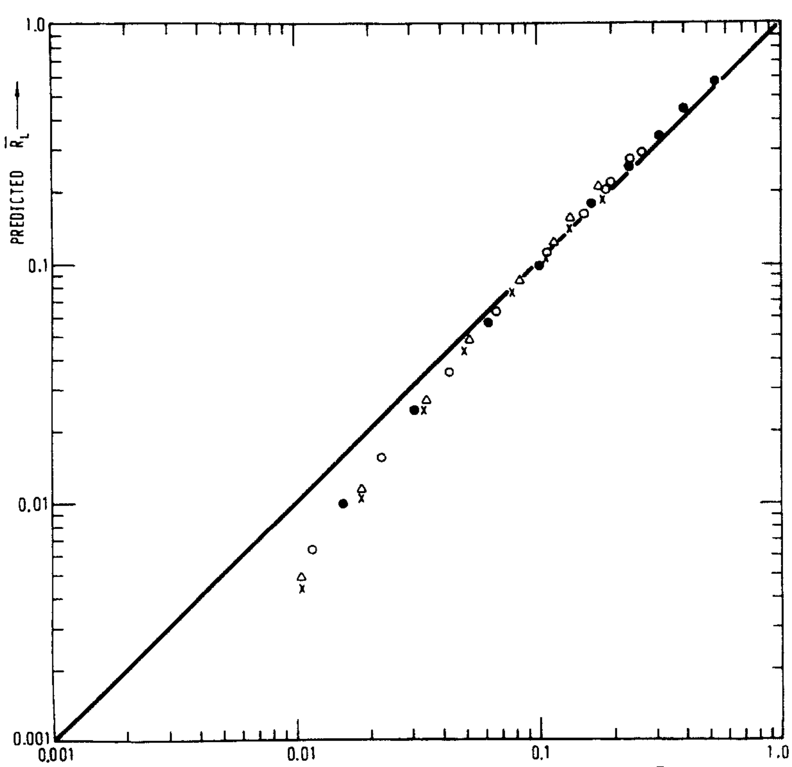


Figure 4. Comparison of the predictions of [6] with the measured $\bar{R}_L$ results of Nguyen & Spedding (1978). Data points as per figure 3.

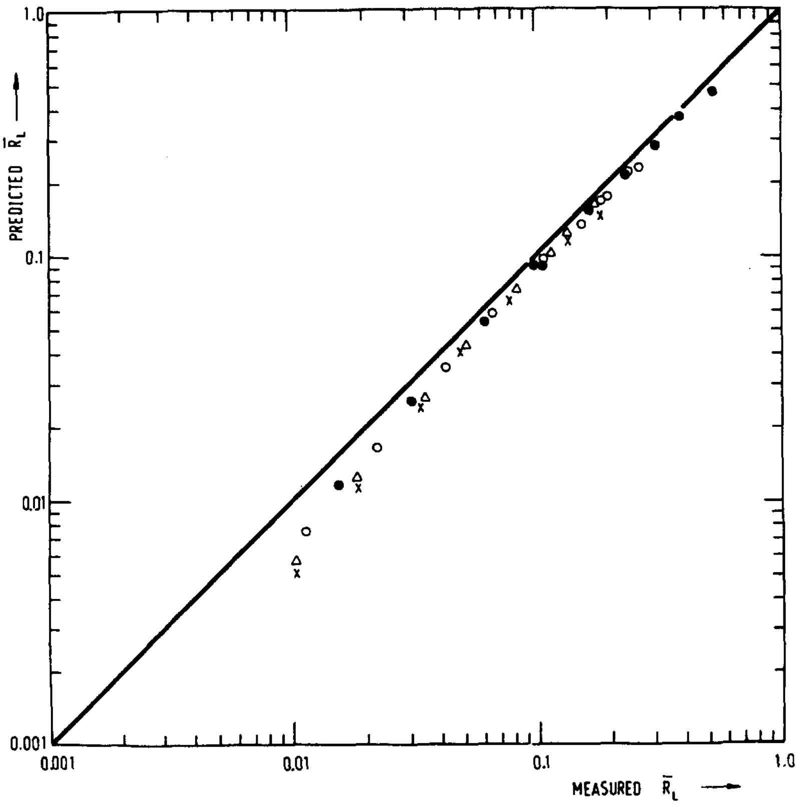


Figure 5. Comparison of the predictions of [4] with the measured $\bar{R}_L$ results of Nguyen & Spedding (1978). Data points as per figure 3.

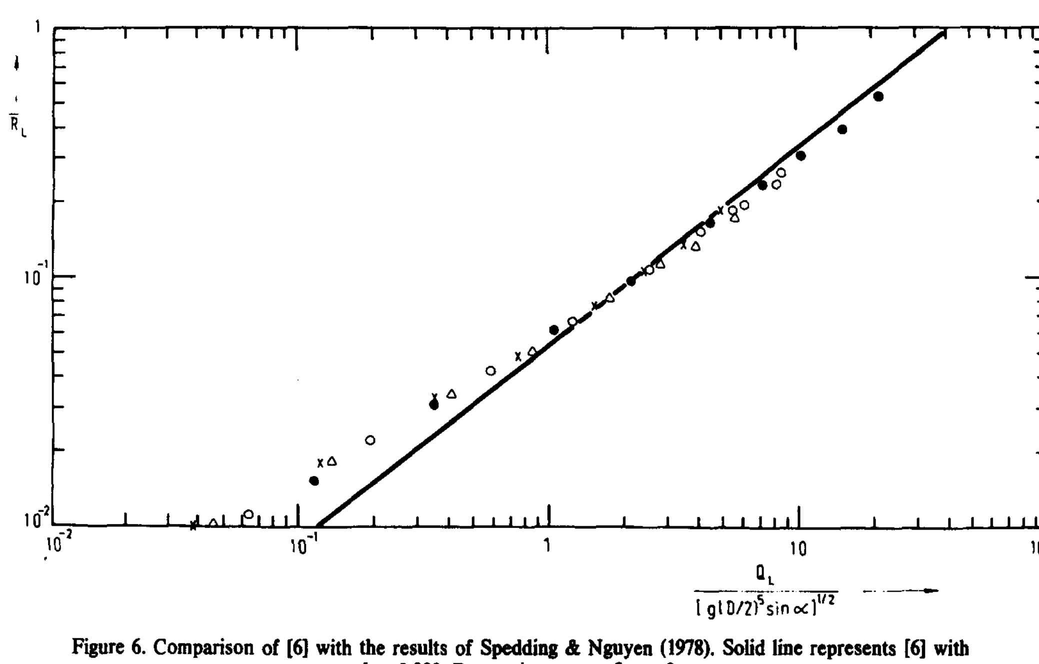


Figure 6. Comparison of [6] with the results of Spedding & Nguyen (1978). Solid line represents [6] with $f_L = 0.008$. Data points as per figure 3.

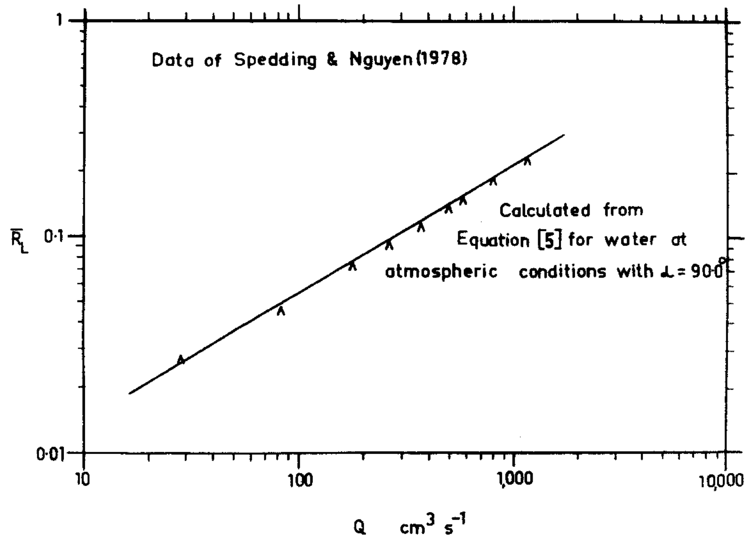


Figure 7. Comparison of predicted $\bar{R}_L$ using [5] against measured $\bar{R}_L$ for $\alpha = 90°$ at various liquid rates.

the case of $0° < \alpha < 90°$. The oscillating effect also is observed to increase in severity with pipe inclination so the observed deviation between $\bar{R}_L$ data and theory would appear to follow as a direct result of the observed increase in oscillation frequency. Since the liquid surface in the flow cross-section is curved it is logical to expect that any increase in the frequency of oscillation would result in a corresponding increase in the wetted perimeter. Indeed inspection of [4] shows that when all else is equal, an increase in $S_L$ must be balanced by an increase in $\bar{R}_L$.

The deviation of data from prediction for the case of $\alpha = 90°$, as shown in figure 7, is opposite to the trend observed for the case of $0° < \alpha < 90°$. This is because the effects to which the deviation of measured $\bar{R}_L$ to theory are attributable are either missing, with regards the meandering and oscillation of the liquid path at low value of $Q_L$, or have been incorporated into the theoretical development, in the case of the dished radial liquid surface profile. The slight negative deviation of experimental data from predicted in fact would have been expected because of the presence of disturbance surface waves on the liquid which, because they are travelling at a faster speed than the bulk of the liquid in the pipe, will result in a smaller flow area being present to transport the same amount of liquid. This phenomena has been reported

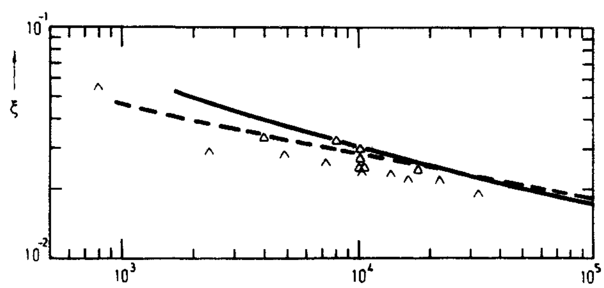


Figure 8. Comparison of data with the method and data (Δ) of Gimbutis *et al.* (1973). Solid line represents Prandtl's equation $1/\sqrt{\xi} = 0.884 \ln \mathrm{Re}\sqrt{\xi} - 0.91$ as given by Gimbutis *et al.* Broken line is given by the equation $\xi = 4 \times f$ where $f$ is given by [3].

by a number of workers for the case of annular flow and was particularly observed by Chen & Spedding (1981).

The results are further shown as plots of $\xi$ against the Reynolds number in figure 8 in accordance with the method of Gimbuties *et al.* (1973) where $\xi$ is defined as being equal to four times $f_L$ of [2]. Since $\xi$ may be shown to be proportional to $\bar{R}_L{}^3$, the deviations become greatly exaggerated. For example, a 6% deviation in figure 7 becomes a 19% deviation in figure 8. Hence it may be stated, from figure 8 that there is reasonable agreement between the data of Nguyen & Spedding (1977) and the small number of data given by Gimbutis *et al.* (1973) for the case of liquid film flow on the inner tube surface.

The method presented in this work can be adapted easily to the situation when the pipe has a rough surface or when the liquid is non-Newtonian in character provided the appropriate equations are used in place of those given by [2] and [3].

## CONCLUSIONS

By assuming a general ideal physical model, equations have been derived for predicting the holdup of liquid draining from an inclined tube. The model has shown substantial agreement with experimental data which were obtained using water in a 4.55 cm diameter perspex pipe at inclinations of $90° \geq \alpha > 0°$. For the case of $90° > \alpha > 0°$, this analysis which allows for the Reynolds number variation is an improvement on the equation derived from the work of Kubie & Gardner (1978), which underpredicts results for $\bar{R}_L < 0.1$ and overpredicts for $\bar{R}_L > 0.1$, and depends on the use of a constant value of $f_L = 0.008$ introduced in an attempt to obtain agreement with experimental results. For the case of $\alpha = 90°$, there is substantial agreement with the results of Gimbutis *et al.* (1973).

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